

Nonsinusoidal current-phase relations and the $0 - \pi$ transition in diffusive ferromagnetic Josephson junctions

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We study the effect of the interfacial transparency on the Josephson current in a diffusive ferromagnetic contact between two superconductors. In contrast to the cases of the fully transparent and the low-transparency interfaces, the current-phase relation is shown to be nonsinusoidal for a finite transparency. It is demonstrated that even for the nearly fully transparent interfaces the small corrections due to weak interfacial disorders contribute a small second-harmonic component in the current-phase relation. For a certain thicknesses of the ferromagnetic contact and the exchange field this can lead to a tiny minimum supercurrent at the crossover between 0 and π states of the junction. Our theory has a satisfactory agreement with the recent experiments in which a finite supercurrent was observed at the transition temperature. We further explain the possibility for observation of a large residual supercurrent if the interfaces have an intermediate transparency.

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I. INTRODUCTION

Ferromagnet-superconductor hybrid structures exhibit novel and interesting phenomena which have been studied extensively in the recent years (for a review see Refs. 1 and 2). These systems provide the possibility for a controlled study of coexistence and competition of the ferromagnetism and the superconductivity. One of the most interesting effects is the possibility of forming the so-called π Josephson junction in superconductor-ferromagnet-superconductor (SFS) structures^{3,4,5,6,7,8,9,10,11,12}. In a π junction the ground-state phase difference between two coupled superconductors is π instead of 0 as in the ordinary 0 junctions¹. The existence of the π junction in layered SFS systems, which was first predicted by Bulaevskii *et al.*,³ and Buzdin *et al.*⁴ to occur for certain thicknesses and exchange field energies of the F layer, has been confirmed in experiment⁹. Ryazanov *et al.*⁹ observed the formation of the π junction in diffusive SFS junctions by measuring nonmonotonic variations of the Josephson critical current on changing the temperature. At the transition from 0 to π coupling the critical current undergoes a sign change and thus passes through a zero-current minimum. This appears as a cusp in the temperature dependence of the absolute values of the critical current which is measured in the experiment. Later the nonmonotonic temperature dependence of the critical current in diffusive SFS contacts was observed in other experiments^{13,14,15,16} and was attributed to the $0 - \pi$ transition induced by the ferromagnetic exchange field. The $0 - \pi$ transition have been studied theoretically by several authors in clean^{8,10,11,12} and diffusive^{5,8,12,17} F Josephson contacts for different conditions and strength of disorder at the FS interfaces. The theory turned out to be in good agreement with experiment¹⁸.

Very recently a new experiment revealed another characteristic of the $0 - \pi$ transition in SFS junctions, which was not detected before. While in the early experiments^{9,13,14,15,16} the critical current at the crossover temperature was found to vanish, Sellier *et al.*¹⁹ reported the existence of a finite small supercurrent at the transition temperature $T_{0\pi}$ in diffusive SFS junctions for a certain thickness of the F layer. A finite critical

current at the crossover was predicted theoretically^{10,11} in ballistic F contacts, and was attributed to the appearance of the second harmonic in the Josephson current-phase relation. In Ref. 19 further analysis of the residual supercurrent by applying high-frequency excitations showed half-integer Shapiro steps, confirming the appearance of the second harmonic in the Josephson current-phase relation. This observation was confirmed by other experiments in Ref. 20, although it was argued that the appearance of the half-integer Shapiro steps in the current-voltage characteristic of the SFS contacts may also originate from a non uniform thickness of the F layer.

For diffusive SFS contacts the theoretical studies mostly concentrated on either the perfectly transparent FS interface or the low-transparency case. For these two limiting cases and in the weak-proximity-effect regime using the linearized Usadel equations²¹ implemented with the Kupriyanov and Lukichev²² (KL) boundary conditions at the FS interfaces, the current-phase relation (CPR) is found to be always sinusoidal. Melin²³ was the first who proposed a theoretical description of the $\sin(2\varphi)$ term of the CPR in SFS junctions in which a strong decoherence in the magnetic alloy can explain the magnitude of the residual supercurrent at the $0 - \pi$ transition. Latter Houzet *et al.*²⁴ and Buzdin²⁵ independently presented an explanation of the nonsinusoidal CPR by considering the effect of magnetic impurities on the Josephson current in diffusive SFS contacts using the KL boundary condition in the highly transparent limit. They showed that the isotropic²⁴ or the uniaxial²⁵ magnetic scattering in the ferromagnet can lead to a second harmonic component in the CPR which is responsible for the finite supercurrent at $T_{0\pi}$. The aim of the present work is to study the effects of the finite FS interface in the full range on the $0 - \pi$ crossover in diffusive SFS contacts. To our best knowledge there have not been studies of diffusive F Josephson junctions with the FS interface having an arbitrary transparency. We show that even a small correction of the Josephson current due to weak disorder at the interfaces can produce a second-harmonic term which leads to a small finite supercurrent at the $0 - \pi$ as was observed in experiment.

In this paper we investigate the effect of the disorder at the FS interface on the Josephson supercurrent in the dif-

fusive F contact between two conventional superconductors. We adopt quasiclassical Green's function method in the diffusive limit implemented by the general boundary conditions of Nazarov²⁶, which allow us to obtain the Josephson current through the contact for an arbitrary strength of the barrier at the FS interfaces. We analyze the $0 - \pi$ transition and the CPR in terms of the exchange field and the thickness of the F contact in the full range of the FS interface transparency. In two limits of high- and low-transparent interfaces the $0 - \pi$ transition occurs on changing the temperature for different exchange fields and thicknesses of the F layer. For a transparent interface the $0 - \pi$ transition does not occur for layers of thickness smaller than the coherence length $\xi = \sqrt{\xi_0 \ell_{\text{imp}}}$ [here $\xi_0 = v_F^{(S)}/\pi\Delta_0(T=0)$ is the superconducting coherence length, $v_F^{(S)}$ is the Fermi velocity in the superconductor, $\Delta_0(T=0)$ is the amplitude of the superconducting order parameter at zero temperature and ℓ_{imp} is the mean free path in the F layer] even for very large exchange fields (throughout this paper we use the system of units in which the Plank and Boltzmann constants $\hbar = k_B = 1$). In contrast to this for a low-transparency interface the transition takes place for thinner contacts if the exchange field is strong enough. In these two limiting cases the current-phase relation is sinusoidal provided that the weak-proximity approximation holds. This implies a zero supercurrent at the $0 - \pi$ transition. We show that the corrections to these results, both in high- and low-transparency cases, produce a second-harmonic term proportional to $\sin(2\varphi)$. While for the low-transparency interfaces the second-harmonic term is so small at it can be neglected, it has some measurable value at the high-transparency limit which has oscillatory behavior as a function of the F thickness and the exchange field. For values of these quantities which lead to a positive value of the second-harmonic, the critical current at the $0 - \pi$ transition has a small finite value. This finding is consistent with the experiment¹⁹, where evidence for the second harmonic was found for highly transparent interface samples.

We further show that for interfaces far from being in the high- on low-transparency regime, the CPR deviates strongly from the standard sinusoidal form. The second-harmonic term at the $0 - \pi$ transition can be as the same order as the first harmonic for an intermediate value of the FS interface transparency, leading to a large residual supercurrent at the transition, provided that the second-harmonic term is positive for the given exchange field and thickness of the contact. We analyze the finite supercurrent at the transition in the full range of the interface's transparency.

In Sec. II, we introduce our model of a diffusive SFS contact. Assuming the weak-proximity effect regime we linearize the Usadel equations and the Nazarov boundary conditions at the FS interfaces with respect to the anomalous component of the quasiclassical Green's function in the ferromagnetic side of the interface. We present an expression for the Josephson current which is valid for an arbitrary barrier strength at the FS interfaces. Section III is devoted to analyzing the $0 - \pi$ transition and the finite transition current in the full range of the parameters. We obtain analytical expressions for the second-harmonic corrections in the two limits of high and low trans-

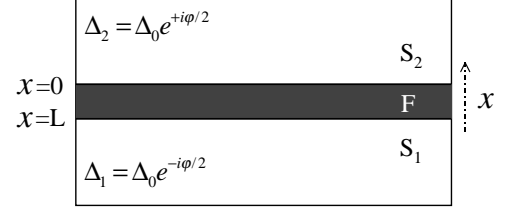


FIG. 1: Schematic of the SFS junction. The current flow between two superconductors ($S_{1,2}$) through a ferromagnetic layer (F). The FS interfaces have a finite transparency.

parencies and also present the results for intermediate transparency. We also compare our results with the finding of the recent experiment. Finally we end with a conclusion in Sec. IV.

II. THE JOSEPHSON CURRENT IN DIFFUSIVE SFS STRUCTURES

In this section we calculate the Josephson current for a diffusive SFS junction in which the FS interfaces have an arbitrary transparency. We consider the SFS junction schematically shown in Fig. 1. It consists of two conventional superconducting reservoirs S_1 and S_2 which are connected by a thin ferromagnetic metal (F) layer. The F layer has a thickness L and is characterized by a homogeneous spin-splitting exchange field (energy) h . The Josephson current through the F layer is determined by the phase difference φ between the superconducting order parameters of the two superconductors. For a diffusive contact the thickness L is much larger than the mean free path of the impurity scattering, ℓ_{imp} . Band mismatches and disorders at the interfaces lead to a backscattering of the quasiparticles. Thus the FS_1 and FS_2 interfaces have finite transparencies which are determined by the interface resistances R_{b_0} and R_{b_L} , respectively. We consider the limit of large exchange fields ($h \gg T_c$), when the superconducting correlations are strongly suppressed upon crossing from the superconducting side to the ferromagnetic side of the FS interfaces. In this case we can assume a weak superconducting proximity effect and apply a theory which is linearized with respect to the induced superconducting order parameter in the F layer.

In the quasiclassical regime ℓ_{imp} is much larger than the Fermi wavelength λ_F which allows us to use the quasiclassical Green's function approach. For the diffusive junction we apply the Usadel equation for the spin-resolved matrix Green's function $\hat{g}_\sigma = \begin{pmatrix} g_\sigma & f_\sigma \\ f_\sigma^\dagger & -g_\sigma \end{pmatrix}$, which in the absence of spin-flip scattering reads

$$[(\omega_n + i\sigma h)\tau_3 + \hat{\Delta}(\mathbf{r}), \hat{g}_\sigma] = D\partial_r \hat{g}_\sigma \partial_r \hat{g}_\sigma. \quad (2.1)$$

Here g_σ and f_σ are, respectively the normal and the anomalous components of the matrix Green's function for spin

σ ($= \pm 1$) electrons which depend on the Matsubara frequency $\omega_n = \pi T(2n + 1)$ and the coordinate \mathbf{r} . The diffusion coefficient $D = v_F^{(F)} \ell_{\text{imp}}/3$, in which $v_F^{(F)}$ is the Fermi velocity in the F layer and τ_i ($i=1,2,3$) denote the Pauli matrices. The matrix $\hat{\Delta}(\mathbf{r}) = \begin{pmatrix} 0 & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & 0 \end{pmatrix}$, in which $\Delta(\mathbf{r})$ is the superconducting pair potential (order parameter). The matrix Green's function \hat{g}_σ satisfies the normalization condition:

$$\hat{g}_\sigma^2 = 1, \quad g_\sigma^2 + f_\sigma f_\sigma^\dagger = 1, \quad (2.2)$$

where $f_\sigma^\dagger(\mathbf{r}, \omega_n, h) = f_\sigma^*(\mathbf{r}, \omega_n, -h)$. We have assumed a thin film structure with thickness L much smaller than the lateral dimensions, so that variations are only along the x axis in the direction perpendicular to the plane of the film.

The current density is obtained from f_σ through the relation

$$j(\varphi) = \frac{i\pi T}{2e\rho_F} \sum_{\sigma=\pm 1} \sum_{\omega_n=-\infty}^{+\infty} \left(f_\sigma \frac{df_\sigma^\dagger}{dx} - f_\sigma^\dagger \frac{df_\sigma}{dx} \right), \quad (2.3)$$

Here ρ_F is the normal-state resistivity of the F layer.

Inside the F layer $\Delta(x) = 0$, but f_σ can still be nonzero due to the proximity effect. We neglect the suppression of the order parameter Δ close to the FS interface inside the superconductors. Thus $\Delta(T) = \Delta_0(T) \exp(\pm i\varphi/2)$ is constant in the superconductors. With this assumption we solve Eq. (2.1) inside the F layer and impose boundary conditions at the two interfaces which relates the values of the Green's functions at the superconducting and ferromagnetic sides of the two interfaces. We apply the general boundary condition developed by Nazarov²⁶, which is valid for an arbitrary value of the barrier height at the interfaces. For \hat{g}_σ the boundary conditions are written as

$$\hat{g}_\sigma \frac{\partial \hat{g}_\sigma}{\partial x} \Big|_{x=0^+(L^-)} = \frac{R_F}{R_{b_{0,L}}} \langle \hat{M}(\theta) \rangle, \quad (2.4)$$

$$\hat{M}(\theta) = \frac{2T_{0,L}(\theta) [\hat{g}_\sigma(0^-(L^-)), \hat{g}_\sigma(0^+(L^+))]}{4 + T_{0,L}(\theta) \{ \hat{g}_\sigma(0^-(L^-)), \hat{g}_\sigma(0^+(L^+)) \} - 2}, \quad (2.5)$$

where R_F is the normal-state resistance of the F layer. The transmission eigenvalues of the interfaces at 0, L are given by $T_{0,L}(\theta) = \cos^2 \theta / (\cos^2 \theta + \gamma_{b_{0,L}})$ with the angle θ showing the direction of the Fermi velocity with respect to the axis normal to the interfaces, $\gamma_{b_{0,L}} = AR_{b_{0,L}}/(\xi\rho_F)$ dimensionless quantities which measure the strength of the scattering from disorders at the interfaces 0, L , respectively, and A the area of the interfaces²⁷. The averaging over directions of the Fermi velocity is defined via

$$\langle \hat{M}(\theta) \rangle = \frac{\int_{-\pi/2}^{\pi/2} d\theta \cos(\theta) \hat{M}(\theta)}{\int_{-\pi/2}^{\pi/2} d\theta \cos(\theta) T(\theta)}. \quad (2.6)$$

[] and { } show the commutator and the anticommutator in $\hat{M}(\theta)$, respectively. Also $g_\sigma(0^-)[f_\sigma(0^-)]$ and

$g_\sigma(0^+)[f_\sigma(0^+)]$ are normal (anomalous) Green's functions at the interface 0 in the superconductor and the ferromagnet sides, respectively. A similar notation holds for $g_\sigma(L^\pm)$ and $f_\sigma(L^\pm)$ at the interface L . These boundary conditions are valid for arbitrary values of $\gamma_{b_{0,L}}$. Equations (2.4)-(2.6) are reduced to the boundary condition of KL in the limit of small transmissions ($\gamma_{b_{0,L}} \gg 1$) and to the condition of the continuity of the Green's function in the limit of high transparencies ($\gamma_{b_{0,L}} \rightarrow 0$).

In a homogeneous bulk s-wave superconductor under the equilibrium condition, the solutions of the Usadel equations are found to be $g_s = \omega_n/\Omega_n$, $f_s = \Delta/\Omega_n$, where $\Omega_n = \sqrt{\Delta_0^2 + \omega_n^2}$.²⁸ In the diffusive limit, the boundary values of the Green's functions in the superconductor sides of the interfaces are approximated by the bulk values:

$$g_\sigma(0^-(L^+)) = g_s = \frac{\omega_n}{\Omega_n}, \quad (2.7)$$

$$f_\sigma(0^-(L^+)) = f_{s1,2} = \frac{\Delta_0}{\Omega_n} e^{\mp i\varphi/2}. \quad (2.8)$$

In general, the Usadel equation (2.1) and the Nazarov boundary conditions Eqs. (2.4)-(2.6) are nonlinear with respect to g_σ and f_σ . We linearize these equations by assuming a weak proximity effect in the F layer which implies $|g_\sigma| \approx 1$ and $|f_\sigma| \ll 1$. With this approximation Eqs. (2.1) and (2.5) are respectively reduced to the following relations

$$(\omega_n + i\sigma h)f_\sigma(x, \omega_n, h) - \frac{D}{2} \frac{\partial^2 f_\sigma(x, \omega_n, h)}{\partial x^2} = 0, \quad (2.9)$$

$$\begin{aligned} \hat{M}(\theta) = & \frac{T_{0,L}(\theta) [\hat{g}_\sigma(0^-(L^-)), \hat{g}_\sigma(0^+(L^+))]}{2 + (g_s - 1)T_{0,L}(\theta)} \\ & \pm \frac{T_{0,L}^2(\theta) f_\sigma(0^-(L^+)) [f_\sigma(0^-(L^-)) f_\sigma^\dagger(0^+(L^+)) + c.c.]}{[2 + (g_s - 1)T_{0,L}(\theta)]^2}. \end{aligned} \quad (2.10)$$

We note that although Eqs. (2.9) and (2.10) are linear with respect to f_σ in the F layer, they are still nonlinear in $T_{0,L}(\theta)$. The anomalous Green's function f_s in the superconducting side of the interface can be of order unity at low temperatures. Thus the FS interfaces have a very small width across which f_σ decreases from the bulk value f_s to f_F (f_F is the anomalous Green's function in the ferromagnet side) which is assumed to be small at large exchange fields $h \gg T_c$.

The solution of Eq. (2.9) inside F layer has the form

$$f_\sigma(x, \omega_n, h) = A_{n\sigma} \cosh(k_{n\sigma} x) + B_{n\sigma} \sinh(k_{n\sigma} x), \quad (2.11)$$

where $k_{n\sigma} = \sqrt{2(\omega_n + i\sigma h)/D}$. Replacing the solution Eq. (2.11) in Eq. (2.3) the expression for the Josephson current is obtained as:

$$I(\varphi) = \frac{i\pi T A}{e\rho_F} \sum_{\sigma=\pm 1} \sum_{n=-\infty}^{+\infty} k_{n\sigma} \text{Im}(A_{n\sigma} B_{n\sigma}^*). \quad (2.12)$$

Here $\rho_F = 1/(2e^2 N_0 D)$, where N_0 is the density of states on the Fermi surface and $Im(\)$ denotes the imaginary part. The coefficients $A_{n\sigma}$ and $B_{n\sigma}$ are derived by applying the boundary conditions (2.4) and (2.10) to the solution (2.11) and using Eqs. (2.7) and (2.8). In this way we have obtained $A_{n\sigma}$ and $B_{n\sigma}$ and replaced the results in (2.12) to obtain the final expression for the Josephson current as

$$I(\varphi) = I_0 \sum_{\sigma=\pm 1} \sum_{n=-\infty}^{n=+\infty} Q_{n\sigma}(\varphi) \sin \varphi, \quad (2.13)$$

where $I_0 = 2\pi T_c / e R_F$, T_c is the superconducting critical temperature, and $Q_{n\sigma}(\varphi)$ is given in the Appendix through Eqs. (A1)-(A7) in which for simplicity we have assumed $\gamma_{b0} = \gamma_{bL} = \gamma_b$.

III. DISCUSSIONS AND RESULTS

Equation (2.13) with $Q_{n\sigma}$ given by Eqs. (A1)-(A7) expresses the Josephson current through the diffusive F layer for an arbitrary value of γ_b . In the limit of $\gamma_b \ll 1$ and $\gamma_b \gg 1$ the expression (2.13) reduces to the results previously obtained for the cases of the fully transparent⁵ and the low-transparency¹⁷ interfaces, respectively. In these two limits the current-phase relation is sinusoidal. Let us first find corrections to these results by expanding the Josephson current Eq. (2.13) in terms of powers of γ_b for small γ_b and keeping the terms up to the order of γ_b . In the opposite limit $\gamma_b \gg 1$ we do expansion in the powers of $1/\gamma_b$ and retain the terms up to the order of $1/\gamma_b^3$. The results in the limits of $\gamma_b \ll 1$ and $\gamma_b \gg 1$ have, respectively, the forms

$$I(\varphi) = I^{(1)} \sin \varphi + I^{(2)} \sin(2\varphi) \quad (3.1)$$

$$I(\varphi) = J^{(1)} \sin \varphi + J^{(2)} \sin(2\varphi). \quad (3.2)$$

Here $I^{(1,2)}$ and $J^{(1,2)}$ are, respectively, the first- and the second-harmonic terms of the CPR in the limits of high and low transparency and are given in Eqs. (A8)-(A13).

Thus we find that a small deviation from the limits $\gamma_b = 0$ and the $\gamma_b \gg 1$ has two effects on the Josephson current. First it contributes a small correction to the first-harmonic current term. Second it produces a small second-harmonic term. The second-harmonics terms $I_{n\sigma}^{(2)}, J_{n\sigma}^{(2)}$ are oscillatory functions of h and L and depend on the temperature T . In spite of their smallness they can be important at the $0 - \pi$ transition where the first-harmonic terms vanish. While in the absence of the second-harmonic terms the critical current undergoes a continuous sign change at the crossover between 0 and π states, for nonvanishing and positive values of $I_{n\sigma}^{(2)}, J_{n\sigma}^{(2)}$ the sign change is discontinuous and the critical current jumps from a positive to a negative value. The discontinuous transition is manifested as a nonzero minimum of the absolute values of the critical current at the $0 - \pi$ transition temperature $T_{0\pi}$.²⁵

In the experimentally relevant limit when $h \gg T_c$ and $L \gtrsim \xi_F$ ($\xi_F = \sqrt{D/\hbar}$ is the superconducting correlation oscillation length in the F layer) we find from Eqs. (A8)-(A10)

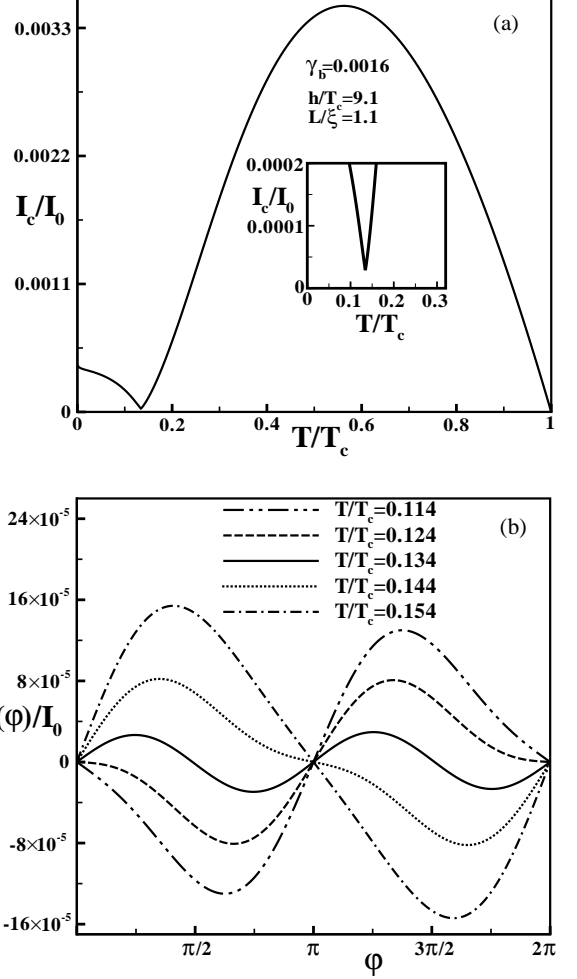


FIG. 2: (a) Dependence of the Josephson critical current on the temperature for $h/T_c = 9.1$, $L/\xi = 1.1$, and $\gamma_b = 0.0016$. Inset shows the temperature dependence near the transition temperature $T_{0\pi}/T_c = 0.134$; $I_c(T)$ has a nonzero minimum at the transition temperature. (b) The current-phase relation for different temperatures around $T_{0\pi}$.

that always $J^{(2)}/J^{(1)} \lesssim 10^{-5}$, so the second-harmonic term in the CPR may be neglected in the low-transparency limit. However for highly transparent interfaces Eqs. (A11)-(A13) gives us that $I^{(2)}/I^{(1)}$ can be as large as 10^{-3} , a ratio that can be observed in experiment.

Let us now compare our results with the findings of the experiment of Ref. 19. In this experiment the temperature dependence of the Josephson critical current was studied in the ferromagnetic alloy layer $\text{Cu}_{1-x}\text{Ni}_x$ with x of the order 0.48, connecting two Nb superconducting electrodes. The normal-resistance measurement of the junctions has shown that the samples have highly transparent FS interfaces. For two different thicknesses of the F layer $L_1 = 17$ nm and $L_2 = 19$ nm, the $0 - \pi$ transition was observed at the temperatures 1.12 and 5.36 K, respectively. For the sample with $L_2 = 19$ nm the critical current had a zero minimum, but

they measured a nonzero supercurrent of about $4 \mu\text{A}$ for the sample of $L_1 = 17 \text{ nm}$ at the $0 - \pi$ transition temperature. The superconducting critical temperature of the samples $T_c = 8.7 \text{ K}$ and the exchange field was estimated to be of the order $h = 5 - 10 \text{ meV}$. In Figs. 2 and 3 we present the Josephson critical current dependence on the temperature and the CPR of the corresponding two samples using our theoretical results given by Eqs. (2.13) and (A8)-(A13). We fix the critical temperature of the system, T_c , the Fermi velocity v_F^S of the superconductors, and the diffusion constant D of the two F layers with thicknesses $L_{1,2} = 17$ and 19 nm , by the values reported in the experiment¹⁹. This leaves two fitting parameters to be obtained, which are the strength of disorder at the interfaces, γ_b , and the exchange field h . To take into account the suppression of the order parameter at the superconducting side of the FS interfaces, which is more pronounced for strong exchange fields $h \gg T_c$ and a highly transparent interface, we have multiplied $\Delta_0(T)$ by a constant factor $\alpha = 0.5$ in our calculations. This, however, does not change the qualitative behavior of I_c with the temperature, but allows for a better fitting with the experimental curves. As in other theoretical studies of the ferromagnetic Josephson junctions (see e.g., Ref. 9) the amplitude of our calculated critical current is substantially ($\sim 10^3$ times) larger than the measured curves. This can be attributed to the strong destruction of the Andreev pairs due to the spin-flip scattering induced by the superparamagnetic behavior of the Cu-Ni alloy¹⁶, which is not taken into account in our calculation.

For the thickness 17 nm the results are presented in Figs. 2(a) and 2(b). We have taken $D = 5 \text{ cm}^2/\text{s}$ and $h/T_c = 9.1$. In Fig. 2(a) the temperature dependence of the critical current is presented for one value of γ_b . There is a good agreement with the experimental curve at $\gamma_b = 0.0016$. Both the functional dependence and the transition temperature are reproduced correctly. At the transition temperature $T_{0\pi}/T_c = 0.134$ the critical current has a finite value $I_c/I_0 \sim 3 \times 10^{-5}$. The finite critical current results from the appearance of the second-harmonic Josephson current in the CPR. This is shown in Fig. 2(b) in which we present the CPR for different temperatures around the transition when $\gamma_b = 0.0016$. Approaching the transition from lower and higher temperatures the CPR deviates from a sinusoidal form and tends to be a purely second harmonic proportional to $\sin(2\varphi)$ at $T_{0\pi}$.

Figure. 3(a) shows the temperature dependence of the critical current for the second thickness 19 nm . The diffusion constant and the exchange field have the values $D = 5 \text{ cm}^2/\text{s}$ and $h/T_c = 7.93$. The value of $\gamma_b = 0.002$ corresponds to the transition observed in the experiment with $T_{0\pi}/T_c = 0.619$ and a zero current minimum. The temperature dependence of the CPR is presented in Fig. 3(b) for $\gamma_b = 0.002$. In contrast to the $L_1 = 17 \text{ nm}$ case the CPR remains almost sinusoidal crossing the transition temperature. At the transition temperature $I(\varphi) = 0$ for all the phases, which represents a continuous $0 - \pi$ transition.

Let us finally analyze the $0 - \pi$ transition at intermediate transparencies. In Fig. 4(a) the temperature dependence of the critical current $I_c(T)$ is presented for different γ_b when $h/T_c = 5$ and $L/\xi = 1.1$. For these values of h and L the

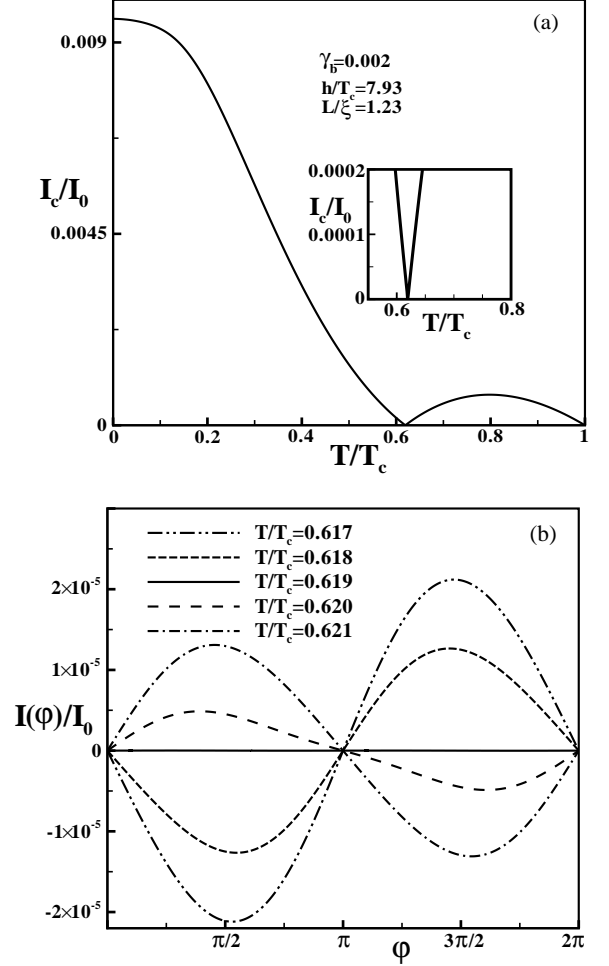


FIG. 3: The same as Fig. 2 but for $h/T_c = 7.93$, $L/\xi = 1.23$, and $\gamma_b = 0.002$.

$0 - \pi$ transition does not take place for fully transparent interfaces, $\gamma_b = 0$. At a lower transparency $\gamma_b = 0.52$ the transition occurs at $T_{0\pi}/T_c = 0.113$ with a large residual minimum supercurrent. On decreasing the transparency, $T_{0\pi}$ increases toward the critical temperature T_c and the transition current decreases and vanishes as $T_{0\pi} \rightarrow T_c$. Thus we find that the highest values of the second-harmonic term can be achieved at the intermediate transparency. This can also be seen from the CPR which is presented in Fig. 4(b) for $\gamma_b = 0.52$ at different temperatures. There is a strong deviation from a sinusoidal CPR relation and $I(\varphi) \propto \sin(2\varphi)$ exactly at the transition temperature. We also obtained that for a fixed thickness by increasing the exchange field the transition temperature $T_{0\pi}$ shifts toward T_c and the minimum current amplitude decreases. Similar behavior holds on increasing the thickness and fixing the exchange field. As a result the $0 - \pi$ transition taking place at a higher temperature has a smaller minimum current. We expect that even for the sample of $L = 19 \text{ nm}$ there is an extremely small minimum supercurrent which was not observable in experiment.

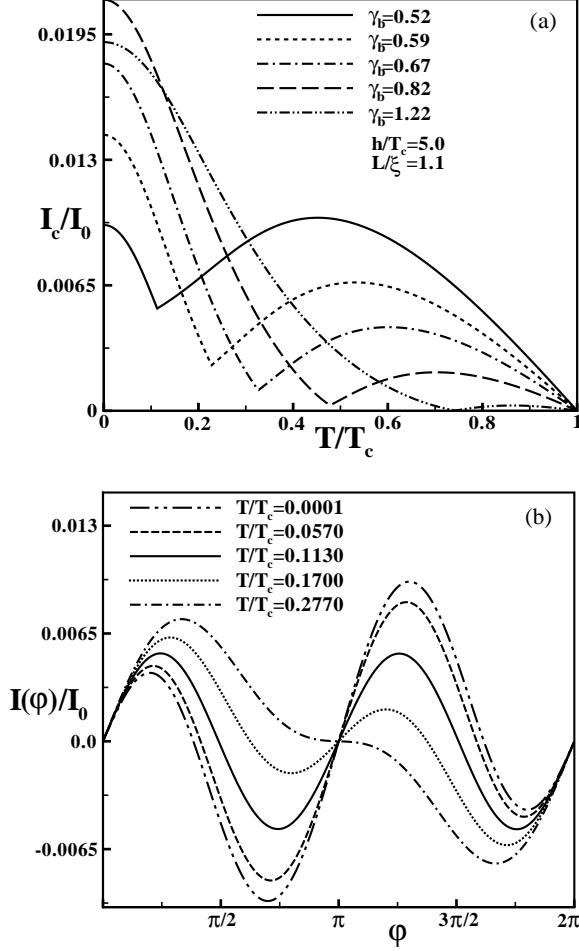


FIG. 4: (a) The Josephson critical current versus T/T_c when $h/T_c = 5$ and $L/\xi = 1.1$, for different γ_b , in the intermediate transparency regime. (b) The current-phase relation dependence on the temperature for $\gamma_b = 0.52$.

IV. CONCLUSION

In conclusion we have explained that the recently observed finite supercurrent at the $0 - \pi$ transition temperature of diffusive ferromagnetic Josephson junctions can originate from a weak quasiparticle backscattering due to small imperfections at the FS interfaces. Using the quasiclassical Green's functions approach in the diffusive limit implemented by the general boundary conditions of Nazarov at the FS interfaces we have obtained an expression of the Josephson current through the F layer which is valid for any strength of the interfacial barrier. At the limiting cases of a perfectly transparent and a low-transparency interface our results reduce to the previously obtained expressions of the Josephson currents with sinusoidal current-phase relations. We have found that in both limits the corrections to these results contain a second-harmonic component. While for the experimentally relevant values of the ferromagnetic layer thickness ($L/\xi_F \gg 1$) and the exchange field ($h/T_c \gg 1$) the second-harmonic correction is

very small and can be neglected in the low-transparency limit, it can have an observably large positive value for highly transparent interfaces. We have demonstrated that in the highly transparent interfaces the positive second-harmonic current results in a finite supercurrent at the $0 - \pi$ transition, which has been observed in experiment. We have compared our results with the experimental curves and found a satisfactory agreement. We also have shown that for an intermediate transparency of the interfaces the second-harmonic Josephson current can be as large as the first harmonic, leading to a large residual supercurrent at the $0 - \pi$ crossover in this limit.

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APPENDIX A: THE EXPRESSION $Q_{n\sigma}$ IN THE JOSEPHSON CURRENT FOR THE CASE $\gamma_{b0} = \gamma_{bL} = \gamma_b$

After deriving the coefficients $A_{n\sigma}$ and $B_{n\sigma}$ and replacing the results in Eqs. (2.12) we obtain the final expression for the Josephson current which is given by Eqs. (2.13), in which $Q_{n\sigma}(\varphi)$ is given by

$$Q_{n\sigma}(\varphi) = \frac{32b^2LT}{T_c} \frac{k_{n\sigma}\beta_1\Delta_0^2}{\beta_0} \times [\beta_1 + 16c\Delta_0^2(-\gamma_b\xi \cos \varphi + \beta_2)]^2, \quad (\text{A1})$$

$$\beta_0 = (\beta_1^2 + 32c\Delta_0^2\{\beta_1\beta_2 + 4c\Delta_0^2[-k_{n\sigma}\gamma_b^2\xi^2 \cos(2\varphi) + \beta_3]\})^2, \quad (\text{A2})$$

$$\beta_1 = \Omega_n^2 [16bg_s k_{n\sigma} \gamma_b \xi \cosh(k_{n\sigma}L) + (64b^2g_s^2 + k_{n\sigma}^2\gamma_b^2\xi^2) \sinh(k_{n\sigma}L)], \quad (\text{A3})$$

$$\beta_2 = k_{n\sigma}\gamma_b\xi \cosh(k_{n\sigma}L) + 8bg_s \sinh(k_{n\sigma}L), \quad (\text{A4})$$

$$\beta_3 = (64b^2g_s^2 + k_{n\sigma}^2\gamma_b^2\xi^2) \cosh(2k_{n\sigma}L) + 64b^2g_s^2(-1 + \frac{k_{n\sigma}\gamma_b\xi}{4bg_s}) \sinh(2k_{n\sigma}L), \quad (\text{A5})$$

$$b = \langle \frac{T(\theta)}{(g_s - 1)T(\theta) + 2} \rangle, \quad (\text{A6})$$

$$c = \langle \frac{T^2(\theta)}{((g_s - 1)T(\theta) + 2)^2} \rangle. \quad (\text{A7})$$

In the limits of $\gamma_b \ll 1$ and $\gamma_b \gg 1$, we can expand the Josephson current Eq. (2.13) in terms of γ_b . In the limit of $\gamma_b \ll 1$, we keep terms up to the order of γ_b . In the opposite limit of $\gamma_b \gg 1$ we do the expansion in powers of $1/\gamma_b$ and retain terms up to the order of $1/\gamma_b^3$. As a result, the Josephson current can be expressed in terms of the first and second harmonics whose coefficients in the limits of $\gamma_b \ll 1$ and $\gamma_b \gg 1$

are $I^{(1,2)}$ and $J^{(1,2)}$, respectively,

$$\begin{aligned} \frac{I^{(1)}}{I_0} &= \sum_{\sigma=\pm 1} \sum_{n=-\infty}^{n=+\infty} \frac{2LTk_{n\sigma}\Delta_0^2 P_1}{T_c \omega_n^2 P_2^2 \sinh(k_{n\sigma}L)}, \\ P_1 &= 1 - \gamma_b \xi R_1 \Delta_0^4 \coth(k_{n\sigma}L), \\ P_2 &= \frac{\Delta_0^2}{(g_s + g_s^2)\Omega_n^2} + 2, \\ R_1 &= \frac{k_{n\sigma} [-8/(\Omega_n^2 \Delta_0^2) + 176g_s(1 + g_s) + R_2]}{4(1 + g_s)^2 \omega_n^4 P_2^4}, \\ R_2 &= 3 + \frac{224(1 + g_s)^2 [1 + 3\omega_n \Omega_n (1 + g_s)/(7\Delta_0^2)]}{\omega_n^2 \Delta_0^2}, \end{aligned} \quad (\text{A8})$$

$$\frac{I^{(2)}}{I_0} = \sum_{\sigma=\pm 1} \sum_{n=-\infty}^{n=+\infty} \frac{-2\gamma_b \xi LT k_{n\sigma}^2 \Delta_0^4}{T_c \omega_n^4 P_2^3 \sinh^2(k_{n\sigma}L)}. \quad (\text{A10})$$

$$\frac{J^{(1)}}{I_0} = \sum_{\sigma=\pm 1} \sum_{n=-\infty}^{n=+\infty} \frac{LT \Delta_0^2 R_3}{2T_c \gamma_b^2 \xi^2 \Omega_n^2 k_{n\sigma} \sinh(k_{n\sigma}L)}, \quad (\text{A11})$$

$$\begin{aligned} R_3 &= 1 - \frac{R_4 \coth(k_{n\sigma}L)}{\gamma_b \xi g_s k_{n\sigma} \Omega_n^2}, \\ R_4 &= -8g_s \Omega_n^2 + \frac{\Delta_0^2 [(1 + g_s) - \Delta_0^2]}{(1 + g_s)^2} + \frac{2g_s (k_{n\sigma} \xi)^{-2}}{\sinh(k_{n\sigma}L)}, \end{aligned} \quad (\text{A12})$$

$$\frac{J^{(2)}}{I_0} = \sum_{\sigma=\pm 1} \sum_{n=-\infty}^{n=+\infty} \frac{-LT \Delta_0^4}{4(1 + g_s) T_c \gamma_b^3 \xi^3 \Omega_n^4 k_{n\sigma}^2 \sinh^2(k_{n\sigma}L)}. \quad (\text{A13})$$

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- ¹ A. I. Buzdin, Rev. Mod. Phys. **77**, 935 (2005).
 - ² A. G. Golubov, M. Yu. Kupriyanov, and E. Il'ichev, Rev. Mod. Phys. **76**, 411 (2004).
 - ³ L. N. Bulaevskii, V. V. Kuzii, and A. A. Sobyanin, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 314 (1977) [JETP Lett. **25**, 290 (1977)].
 - ⁴ A. I. Buzdin, L. N. Bulaevskii, and S. V. Panyukov, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 147 (1982) [JETP Lett. **35**, 178 (1982)].
 - ⁵ A. I. Buzdin and M. Yu. Kupriyanov, Pis'ma Zh. Eksp. Teor. Fiz. **53**, 308 (1991) [JETP Lett. **53**, 321 (1991)].
 - ⁶ A. Buzdin, Phys. Rev. B **62**, 11377 (2000).
 - ⁷ M. Zareyan, W. Belzig, and Yu. V. Nazarov, Phys. Rev. Lett. **86**, 308 (2001).
 - ⁸ F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Phys. Rev. B **64**, 134506 (2001).
 - ⁹ V. V. Ryazanov, V. A. Oboznov, A. Yu. Rusanov, A. V. Veretennikov, A. A. Golubov, and J. Aarts, Phys. Rev. Lett. **86**, 2427 (2001).
 - ¹⁰ N. M. Chtchelkatchev, W. Belzig, Yu. V. Nazarov, and C. Bruder, Pis'ma Zh. Eksp. Teor. Fiz. **74**, 357 (2001) [JETP Lett. **74**, 323 (2001)].
 - ¹¹ Z. Radovic, N. Lazarides, and N. Flytzanis, Phys. Rev. B **68**, 014501 (2003).
 - ¹² N. M. Chtchelkatchev, Pis'ma Zh. Eksp. Teor. Fiz. **80**, 875 (2004) [JETP Lett. **80**, 743 (2004)].
 - ¹³ V. V. Ryazanov, V. A. Oboznov, A. V. Veretennikov, and A. Yu. Rusanov, Phys. Rev. B **65**, 020501(R) (2001).
 - ¹⁴ T. Kontos, M. Aprili, J. Lesueur, F. Genêt, B. Stephanidis, and R. Boursier, Phys. Rev. Lett. **89**, 137007 (2002).
 - ¹⁵ W. Guichard, M. Aprili, O. Bourgeois, T. Kontos, J. Lesueur, and P. Gandit, Phys. Rev. Lett. **90**, 167001 (2003).
 - ¹⁶ H. Sellier, C. Baraduc, F. Lefloch, and R. Calemczuk, Phys. Rev. B **68**, 054531 (2003).
 - ¹⁷ A. Buzdin, Pis'ma Zh. Eksp. Teor. Fiz., **78**, 1073 (2003) [JETP Lett. **78**, 583 (2003)].
 - ¹⁸ A. Cottet and W. Belzig, Phys. Rev. B **72**, 180503(R) (2005).
 - ¹⁹ H. Sellier, C. Baraduc, F. Lefloch, and R. Calemczuk, Phys. Rev. Lett. **92**, 257005 (2004).
 - ²⁰ S. M. Frolov, D. J. Harlingen, V. V. Bolginov, V. A. Oboznov, and V. V. Ryazanov, cond-matt/0506003 (unpublished).
 - ²¹ K. D. Usadel, Phys. Rev. Lett. **25**, 507 (1970).
 - ²² M. Yu. Kupriyanov and V. F. Lukichev, Zh. Eksp. Teor. Fiz. **94**, 139 (1988) [Sov. Phys. JETP **67**, 1163 (1988)].
 - ²³ R. Mélin, Europhys. Lett., **69**, 121 (2005).
 - ²⁴ M. Houzet, V. Vinokur, and F. Pistolesi, Phys. Rev. B **72**, 220506(R) (2005).
 - ²⁵ A. Buzdin, Phys. Rev. B **72**, 100501(R) (2005).
 - ²⁶ Yu. V. Nazarov, Superlattices Microstruct. **25**, 1221 (1999).
 - ²⁷ G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982); see also Y. Tanaka, A. A. Golubov, and S. Kashiwaya, *ibid.* **68**, 054513 (2003).
 - ²⁸ J. Aarts, J. M. E. Geers, E. Brück, A. A. Golubov, and R. Coehoorn, Phys. Rev. B **56**, 2779 (1997).